Trigonometry Workbook

Graphing the trig functions

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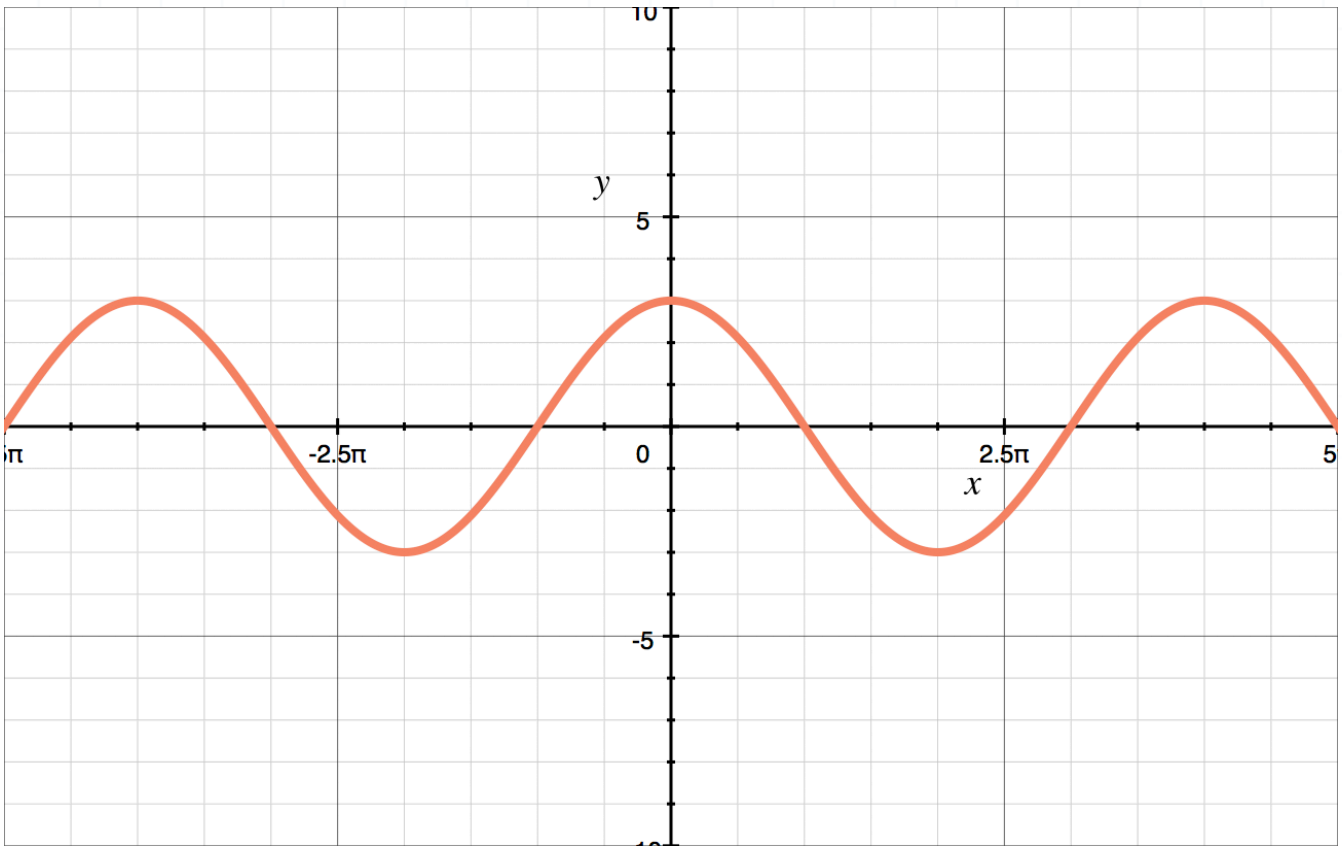
MATH

PERIOD AND AMPLITUDE

■ 1. Find all possible sine and cosine functions $a\sin(bx)$ and $a\cos(bx)$ which have the period of $135°$ and an amplitude of 10.

■ 2. Find the equation of the cosine function.



■ 3. Chuck rides a Ferris wheel at a carnival. The wheel has a 24-meter diameter, and turns at one revolution every 3 minutes. Chuck knows that the altitude of his cabin relative to the center of the wheel is modeled by

$$h(t) = a\cos(bt)$$

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where $t \geq 0$ is the time in minutes, and $t = 0$ is the time when Chuck started to ride. Find a and b to complete the equation for $h(t)$.

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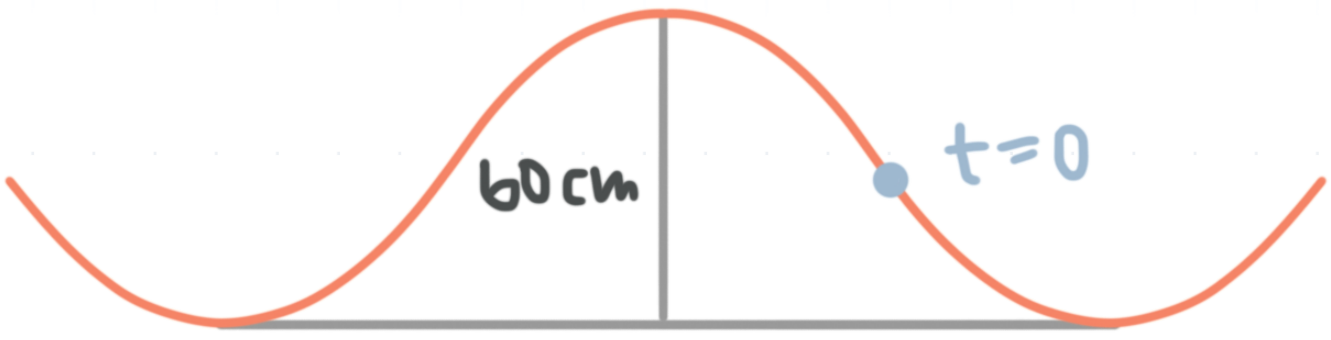
SKETCHING SINE AND COSINE

1. Using set notation, find all values where $f(x)$ is positive.

$$f(x) = -3\sin 2x$$

2. Lisa rested on the seafront. She noted that the waves have a height of 60 cm from trough to crest (from the lowest to highest point), and that they occur every 8 s. Write an equation that models the height of a particle in water in the form $h(t) = a\sin(bt)$ where t is the time in seconds, and $h(t)$ is the height above the sea level in meters. Assume that $t = 0$ is the moment when a particle is in the middle of the front side of the wave. Find a and b and sketch the graph of the function.



3. Donald rides a ferris wheel at a carnival. The altitude of his cabin relative to the center of the wheel is modeled by $h(t) = a\cos(bt)$, with $t \geq 0$, where t is time in minutes. Assume that $t = 0$ is the moment when Donald entered his cabin at the bottom of the wheel. The wheel has a 10-meter

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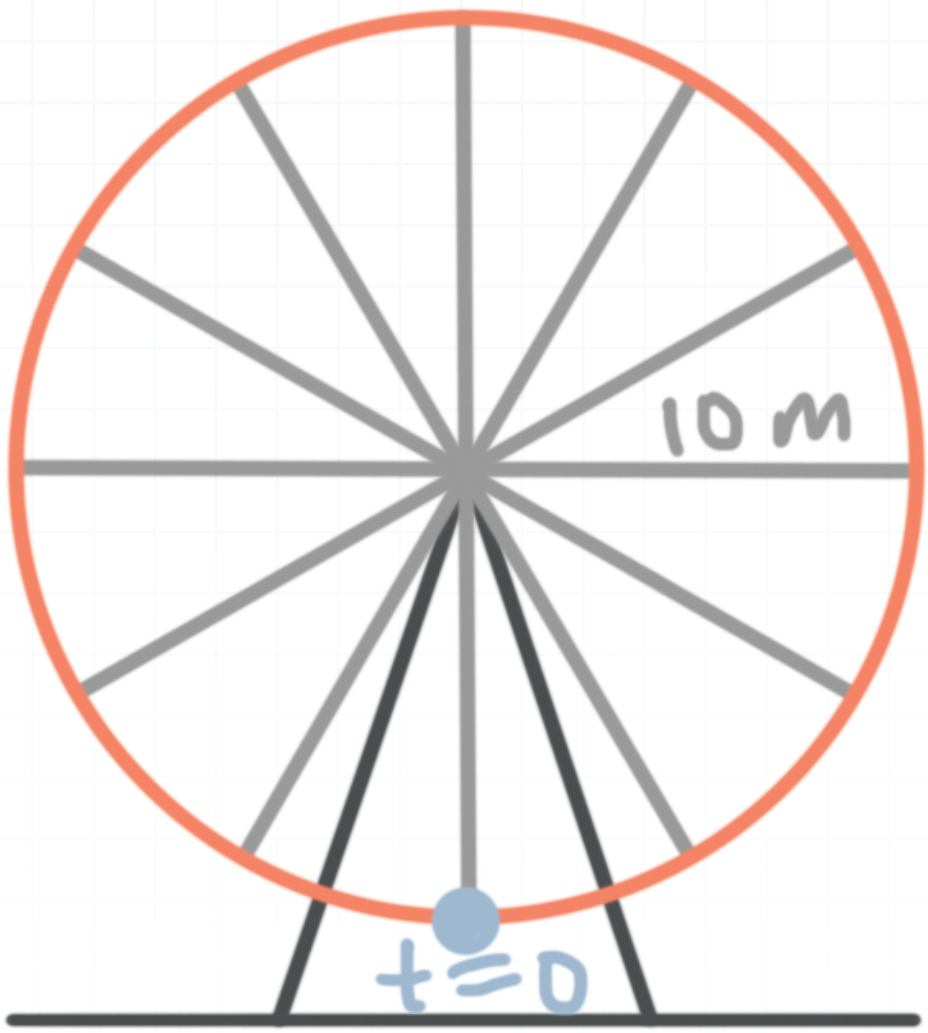
radius, and turns at one revolution every two minutes. Find a and b, and sketch the graph of the function.



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HORIZONTAL AND VERTICAL SHIFTS

■ 1. The solutions of the trigonometric equation $f(x) = 0$ are $x \in \{\pi + 3\pi k : \ k \in \mathbb{Z}\}$. Find the set of solutions for each equation.

(a) $f\left(x + \frac{\pi}{2}\right) = 0$

(b) $f(x - 5\pi) = 0$

■ 2. Find all values where $y = \sin(x) + 1$ is positive.

■ 3. Tom and Jerry ride a 12-cabin Ferris wheel at a carnival. Jerry sat in the first cabin, but Tom was late and sat in the second. The altitude of Jerry's cabin with respect to the center of the wheel is modeled by $h(t) = -6\cos(t)$ with $t \geq 0$, where t is the time in minutes, and $t = 0$ is the moment when Jerry gets in his cabin. Find the equation that models the altitude of Tom's cabin and sketch the graph of the function.

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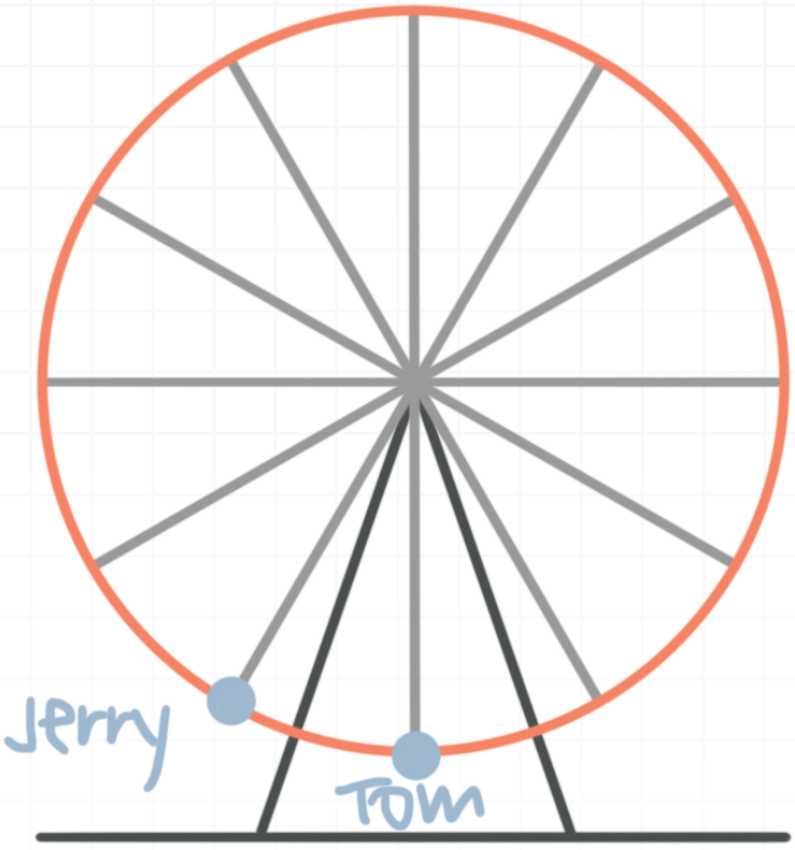

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SEQUENCE OF TRANSFORMATIONS

■ 1. What are the period, amplitude, domain, and range of the function?

$$y = -3\cos(2\pi x - 1) + 4$$

■ 2. Find the equation of $f(x) = \sin(x - \pi)$ after the sequence of transformations:

A horizontal compression by 2, then

A horizontal shift to the right by 3π, then

A vertical stretching by 5, then

A vertical shift down by 2, then

A reflection about the x-axis.

■ 3. Find the set of angles x where the function reaches its maximum value.

$$f(x) = 3\sin\left(\frac{x}{2} + \frac{\pi}{4}\right)$$

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GRAPHING COMBINATIONS OF FUNCTIONS

■ 1. What is the period of the function?

$$f(x) = 2\sin\left(\frac{2x}{3}\right)\left(\cos(x-\pi) + 4\sin\left(\frac{x}{4}\right)\right)$$

■ 2. Find the zeros of the function.

$$f(x) = \frac{\sin(\pi x - \pi)}{\sin\left(\frac{\pi x}{2} + 3\pi\right)}$$

■ 3. We know that the graph of $f(x) = \sqrt{3}\sin x + \cos x$ sketched out in the graph below is identical to the graph of some sine function $g(x) = a\sin(bx + c)$. Find $g(x)$.

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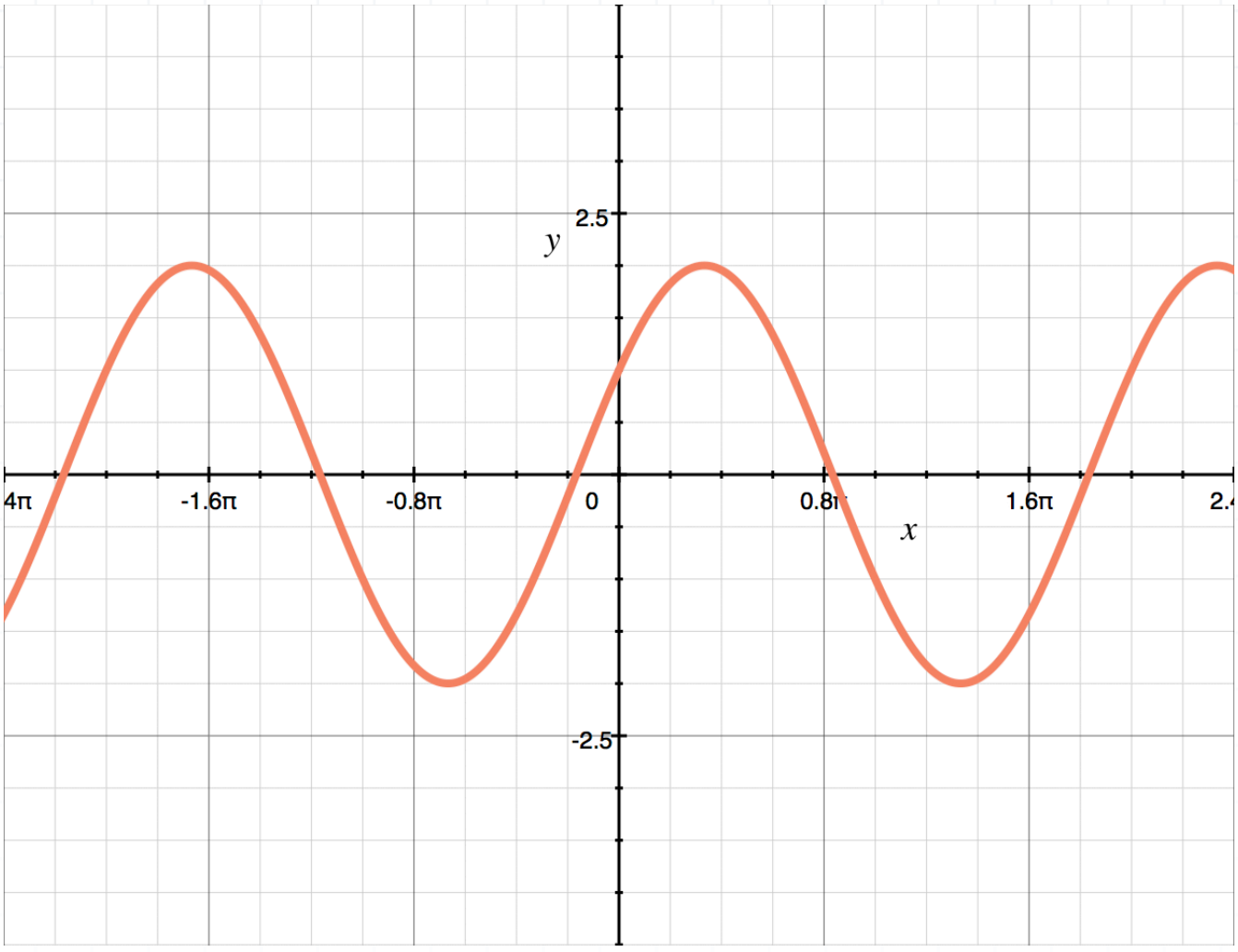

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TRANSFORMATIONS ALGEBRAICALLY

■ 1. The function $g(x)$ is made from $f(x)$ by applying a sequence of transformations:

Horizontal shift to the right by 3, then

Horizontal stretching by 2, then

Horizontal shift to the left by 4, then

Vertical compression by 2.

Find the zeros of the function $g(x)$.

$$f(x) = \frac{x^3 - 3x^2}{2x^2 + 5}$$

■ 2. Find the sequence of transformations needed to transform $f(x)$ into the basic parabola $y = x^2$.

$$f(x) = 2x^2 - 8x + 3$$

■ 3. For $f(x) = -x^2 + 4x$, find its reflection:

Across the x-axis, then

Across the y-axis, then

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About the origin, and then

Across the line $y = x$

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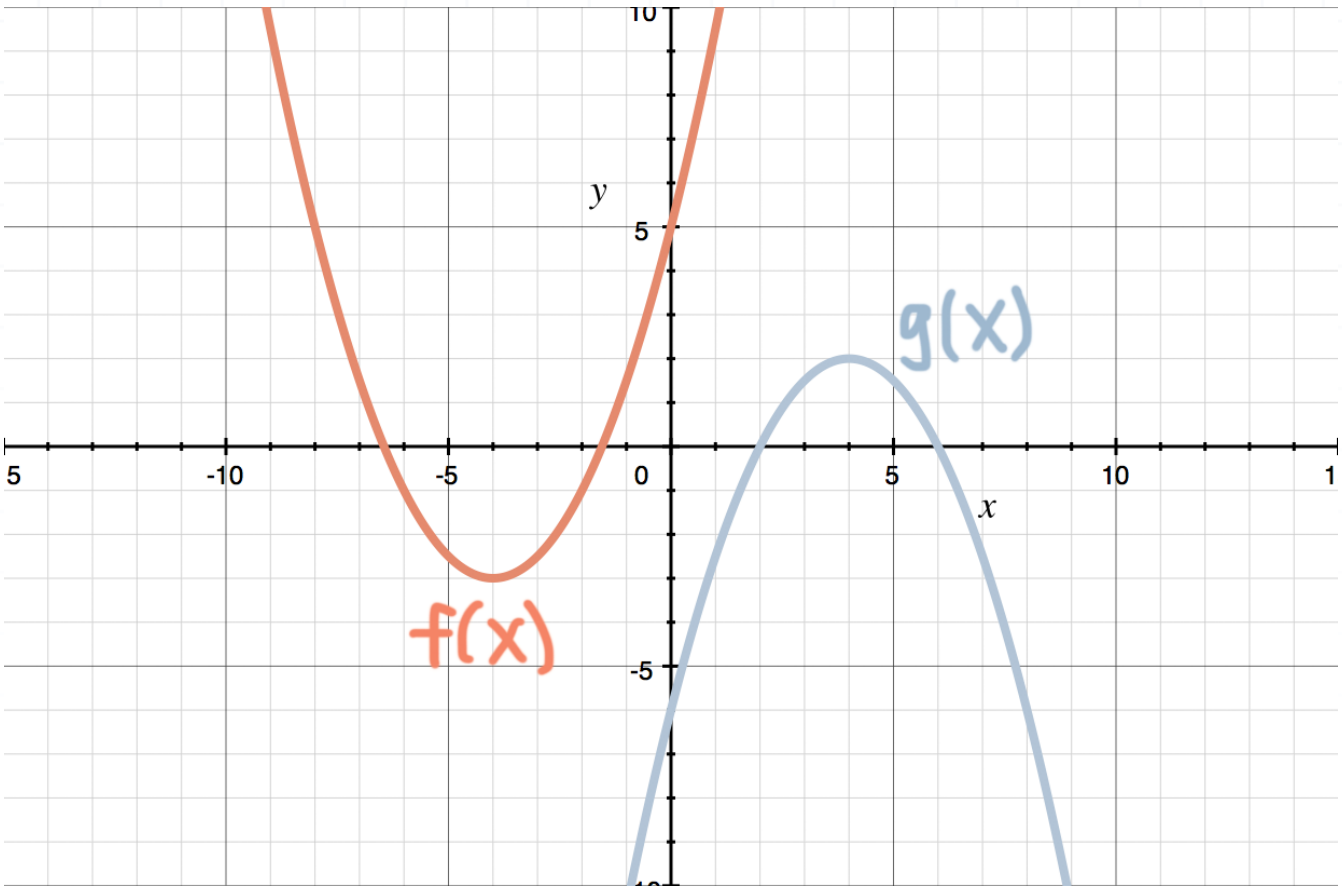
SKETCHING GRAPHS OF TRANSFORMATIONS

■ 1. Find the minimum number of transformations (shifts, compressions, stretching, reflection, about the axes and the origin) required to change $f(x)$ into $g(x)$.



■ 2. Sketch the graph of the piecewise function $f(x)$ shown in the graph after the sequence of transformations:

A vertical compression by 2, then

A horizontal shift to the right by 1, then

A reflection about the y-axis, then

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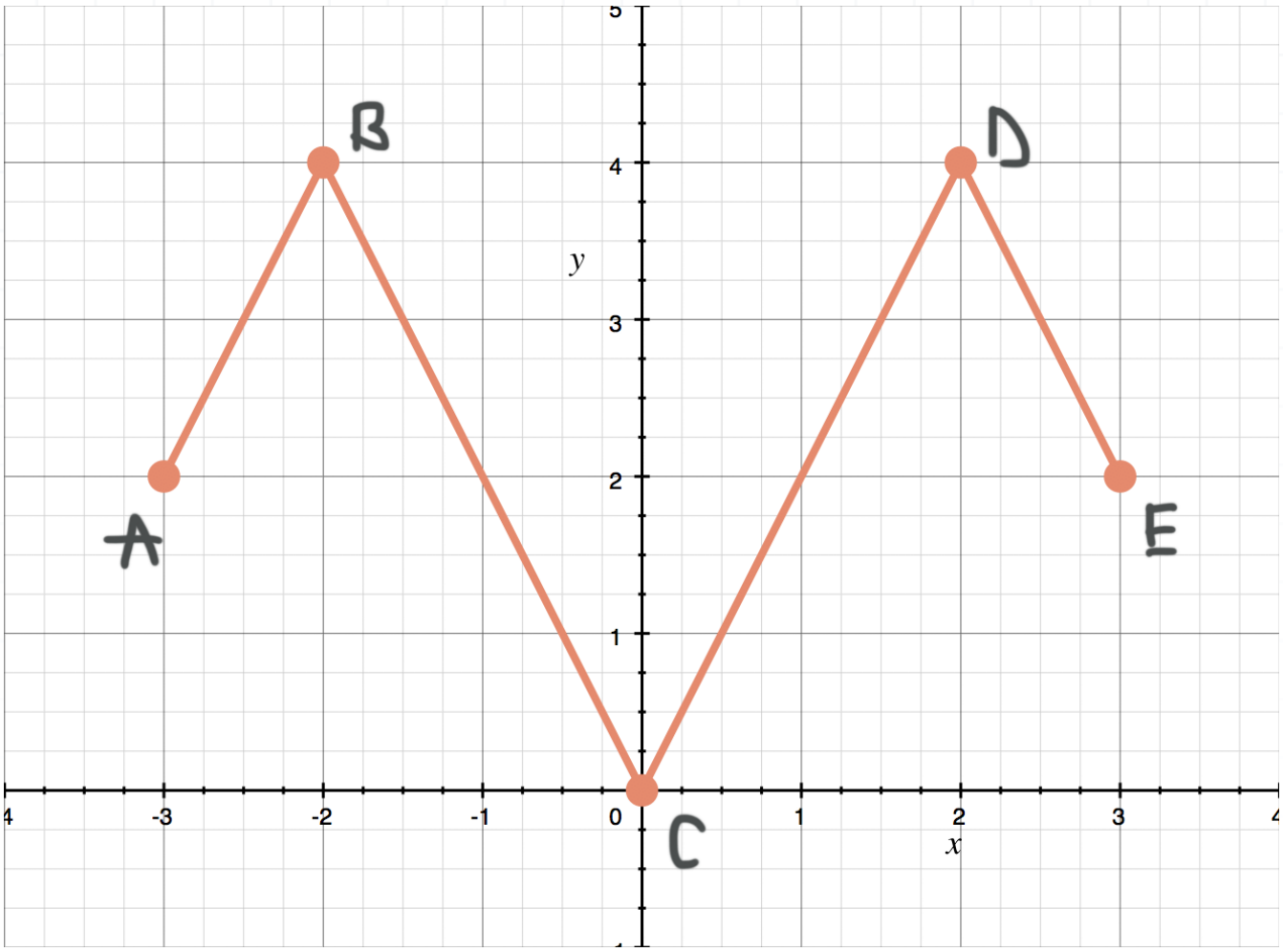
A vertical stretching by 2, and then

A horizontal shift to the left by 1.

The sketch of the function is



■ 3. Find the equation and sketch the graph of the function obtained by compressing the graph of $f(x) = x^2 - 4x + 10$ with respect to the line $y = 3$ by a factor of 3.

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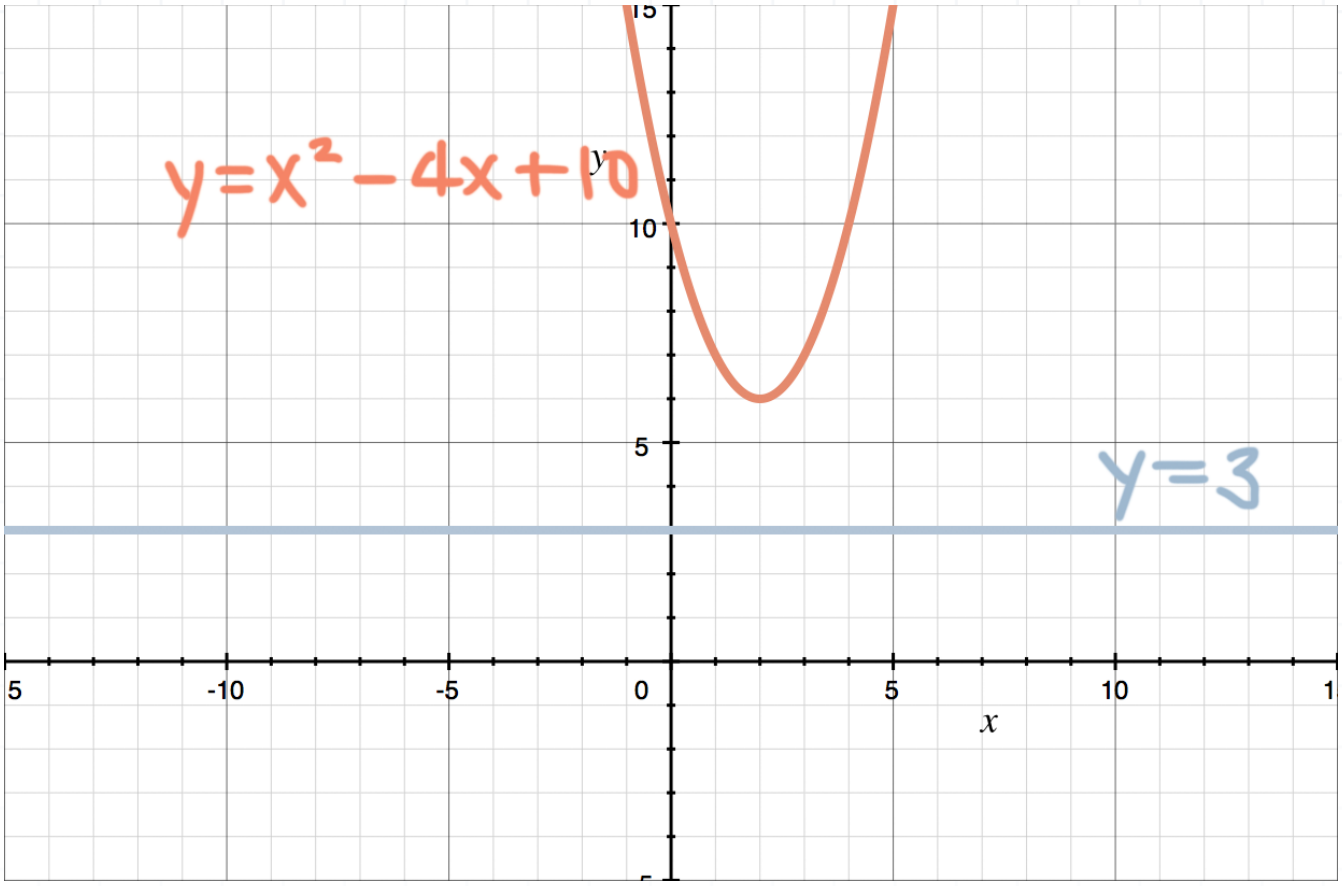

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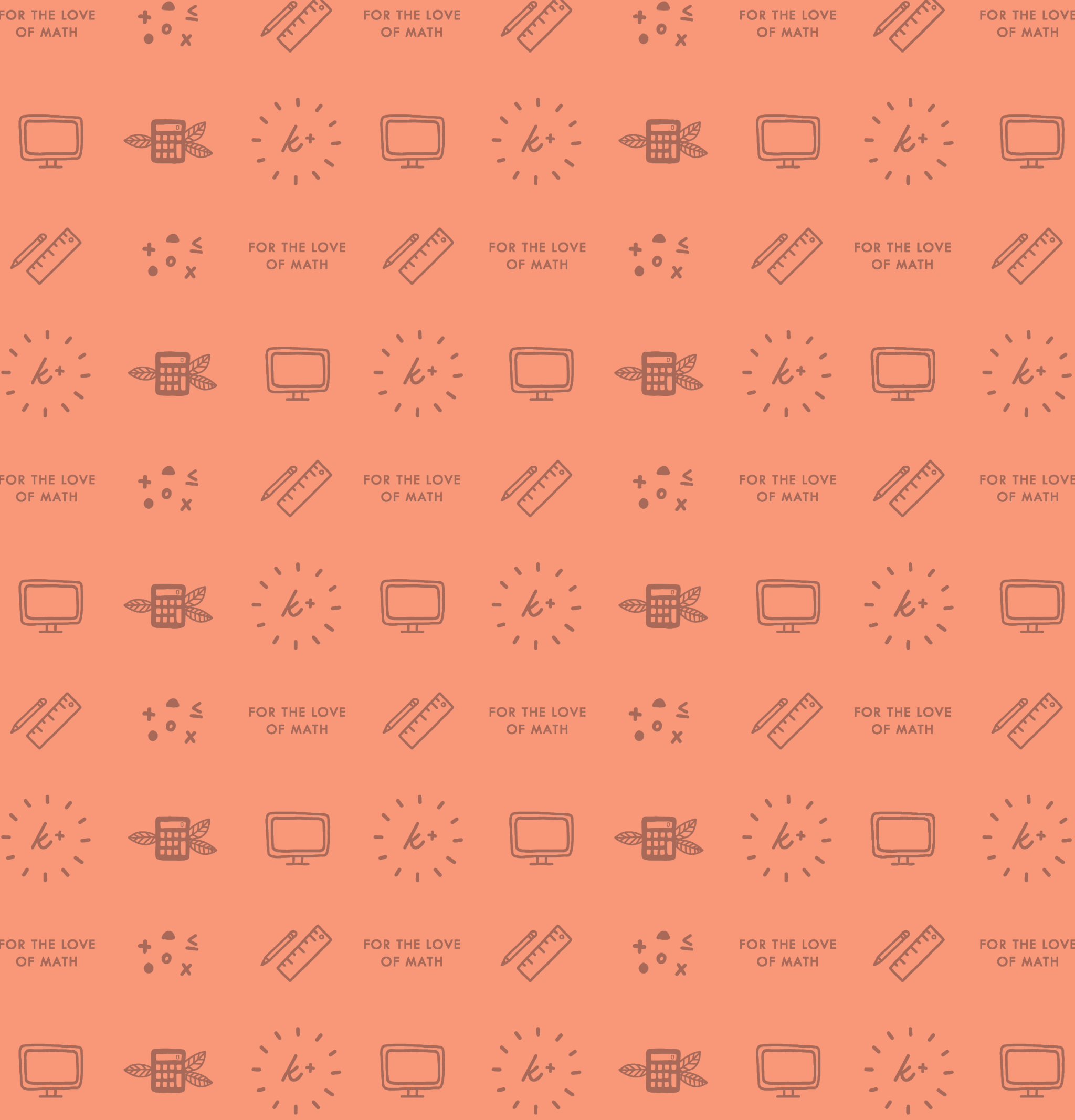
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